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Turbulent electron transport in edge pedestal by electron temperature gradient turbulence

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We present a model for turbulent electron thermal transport at the edge pedestal in high (H)-mode plasmas based on electron temperature gradient (ETG) turbulence. A quasi-linear analysis of electrostatic toroidal ETG modes shows that both turbulent electron thermal diffusivity and hyper-resistivity exhibits the Ohkawa scaling in which the radial correlation length of turbulence becomes the order of electron skin depth. Combination of the Ohkawa scales and the plasma current dependence results in a novel confinement scaling inside the pedestal region. It is also shown that ETG turbulence induces a thermoelectric pinch, which may accelerate the density pedestal formation. © *2013 AIP Publishing LLC*. [http://dx.doi.org/10.1063/1.4829673]

I. INTRODUCTION

Understanding turbulent transport of magnetically confined plasmas is the major scientific effort in fusion plasma physics. In particular, elucidation of the transport process occurring in the edge transport barriers (edge pedestal) in high (H)-mode plasmas[1] is an urgent issue in contemporary fusion research due to the impact of the pedestal on fusion performance. Presently, H-mode operation is the baseline plasma scenario in International Thermonuclear Experimental Reactor (ITER).[2] Turbulent transport in magnetically confined plasmas is driven by micro-instabilities. For the past decades, significant progress has been made in understanding the physics of ion thermal transport channel in magnetically confined plasmas. In contrast to these achievements, however, some aspects of turbulent transport in electron and particle channels still remain to be elucidated. Representative candidates as possible drivers for turbulent electron thermal transport include the electrostatic trapped electron mode (TEM)[3,4] and the electron temperature gradient (ETG) mode.[5,6] In terms of the mixing length argument, the radial correlation length for TEM is the order of ion Larmor radius (ρ_i) scale, while that of the ETG mode is electron Larmor radius (ρ_e).

In H-mode plasmas, drift instabilities on ρ_i scales, such as ion temperature gradient modes (ITG) and TEM, are quenched due to the development of strong $E \times B$ shear[7] as the edge pedestal develops. The ETG mode is then likely to be the dominant turbulence driver giving rise to residual turbulence in the edge pedestal. Besides ETG turbulence, kinetic ballooning modes (KBM)[8,9] have been invoked as a possible turbulence driver in the edge pedestal. In the edge pedestal, both modes may co-exist when the onset condition of KBM is met. We will discuss a possible implication of this in Sec. IV. The main goal of this Letter is to elucidate electron transport in the edge transport barrier due to ETG turbulence.

Anomalous electron transport has shown some puzzling phenomena for the past years, dated back to early tokamak experiments. To explain the Alcator scaling of the energy confinement time (τ_E) in which τ_E in Ohmically heated tokamak plasmas is proportional to the operating density, Ohkawa first proposed a conjecture that radial electron thermal transport may follow the electron skin depth (i.e., $\lambda_s = c/\omega_{pe}$, ω_{pe} the electron plasma frequency, c the speed of light) scale, rather than ρ_e scale, due to some electromagnetic effects.[10] An attempt to elucidate the physics of λ_s scale transport was made by Horton *et al.* based on ETG turbulence.[6] In this model, the appearance of the radial correlation length approaching to λ_s scale is attributed to the inverse cascade process in the high frequency, nonlinear electromagnetic regime. More recently, nonlinear gyrokinetic ETG simulations have shown the presence of radially extended elliptical cells—like streamers, whose radial length scale is much longer than the poloidal correlation length (i.e., $\Delta_x \gg \Delta_y \geq \rho_e$).[11,12] They can enhance radial electron transport significantly,[11] thereby providing a potential explanation of anomalous electron transport in tokamak plasmas in accordance with experimental observations. The other puzzling phenomenon also comes from gyrokinetic ETG simulations. In these simulations, a sudden jump of electron thermal conductivity has been observed in magnetic shear scans, without noticeable changes of linear characteristics of ETG modes.[13,14]

Interestingly, electromagnetic effects (such as magnetic flutter) are reported to be weak in these gyrokinetic ETG simulations. A natural question is then whether electrostatic ETG turbulence can give rise to electron thermal transport featuring correlations longer than ρ_e and sometimes showing a jump of electron heat flux. The goal of this paper is to provide a simple theoretical answer for this question and to discuss its implications to contemporary experiments.

In this paper, we make analytical quasilinear analyses of the ETG mode to evaluate electron thermal and particle transport in the pedestal of H-mode plasmas. We show that the Ohkawa scaling of electron thermal transport arises under certain conditions, which are relevant to edge pedestal region. We find that a jump of electron thermal conductivity is possible near the transition condition where electron transport scaling undergoes a change from ρ_e to λ_s scales. Our calculations also show possible roles of ETG turbulence in pedestal dynamics, including edge localized mode (ELM) bursts and the acceleration of density pedestal formation.

The rest of the paper is organized as follows. In Sec. II, we describe the basic model equations. In particular, we calculate the nonadiabatic ion and impurity responses due to wave-particle resonant interactions. In Sec. III, we derive quasilinear transport coefficients driven by ETG turbulence. It will be shown that ETG turbulence yields a novel confinement scaling in the pedestal. A thermoelectric particle pinch driven by ETG turbulence is also discussed. We conclude this paper in Sec. IV with a brief summary of main results and some discussions.

II. MODEL EQUATIONS

The main goal of this paper is to calculate quasi-linear electron thermal and particle fluxes driven by ETG modes under the assumption that ρ_i-scale modes are suppressed by strong $\vec{E}\times\vec{B}$ shear. To perform these calculations, we re-examine the linear characteristics of the ETG mode in the edge pedestal of H-mode plasmas. ETG modes satisfy the following wave number and frequency ordering: $k_x < k_y$, $k_y\rho_e \le 1$, $\rho_e < k_x^{-1}$, $k_\perp c_i \ge |\omega| \sim \omega_* > k_{||}c_e$, where x, y represent radial and poloidal flux tube co-ordinates, k_x, k_y, $k_{||}$ are the radial, poloidal, and parallel (along the magnetic field) wave vectors, ω is the characteristic frequency of the mode, ρ_j is the Larmor radius of the species j, c_j is the thermal velocity, and Ω_j is the cyclotron frequency: $\rho_j = c_j/\Omega_j$, $c_j = \sqrt{T_j/m_i}$, $\Omega_j = eB/m_jc$, $j = (e, i, I)$, $\omega_* = k_y v_{*e}$, $v_{*e} = k_y\rho_e c_e/L_n$, and $\omega_{*T} = \eta_e\omega_*$ with $\eta_e = L_n/L_{Te}$ and $L_n^{-1} = |-d\ln n/dr|$.

A. Ion and Impurity dynamics

We start our analysis from the ion dynamics in ETG regime. Usually, ions are un-magnetized and adiabatic in ETG dynamics because the perpendicular phase velocity of the ETG mode is smaller than the ion thermal velocity (i.e., $\omega/k_\perp < c_I \sim c_i$). This condition is well satisfied in L-mode and in core regions of H-mode plasmas where the density profiles are relatively flat. In the edge pedestal, however, the density scale length is a few percent of the minor radius (i.e., $L_n \approx 0.03\,$a). Then, ions and impurities can resonate with ETG modes, resulting in the deviation from the adiabaticity condition.

We consider un-magnetized and collisionless ion and impurities in ETG dynamics. In the limit $k_\perp c_{i,I} \sim |\omega|$, ETG mode resonates with background ions, which results in deviation of ions from Boltzmann condition. This non-adiabatic ion response can be determined by drift kinetic equation

$$\frac{\partial\tilde{f}_j}{\partial t} + \vec{V}_\perp\cdot\frac{\partial\tilde{f}_j}{\partial\vec{x}} + \frac{Ze}{m_J}\delta E_\perp\cdot\frac{\partial f_{0j}}{\partial\vec{V}} = 0. \tag{1}$$

Assuming Maxwellian equilibrium distribution functions for ions and impurities in one dimension, $f_{0i} = n_{i0}(m_i/2\pi T_i)^{1/2}\exp(-V_y^2/V_{thi}^2)$, and $f_{0I} = n_{I0}(m_I/2\pi T_I)^{1/2}\exp(-V_y^2/V_{thI}^2)$, where $V_{thi}^2 = 2T_i/m_i$, $V_{thI}^2 = 2T_I/m_I$; and substituting them into Eq. (1), the fluctuating ion and impurity distributions are given by

$$\tilde{f}_i = -\tau_i\tilde{\phi}\frac{V_y}{V_y - \omega/k_y}f_{0i}, \tag{2}$$

$$\tilde{f}_I = -\tau_I Z\tilde{\phi}\frac{V_y}{V_y - \omega/k_y}f_{0I}. \tag{3}$$

The ion density fluctuation can be written as

$$\begin{aligned}\tilde{n}_i &= \frac{1}{n_{i0}}\int\tilde{f}_i\,dV_\perp\\ &= -\frac{\tau_i\,\tilde{\phi}}{\pi^{1/2}V_{thi}}\int dV_y\frac{V_y}{V_y - \omega/k_y}\exp(-V_y^2/V_{thi}^2).\end{aligned} \tag{4}$$

We introduce the following dimensionless parameters:

$$\hat{\omega} = \frac{\omega}{k_y V_{thi}}, \quad \zeta = \frac{V}{V_{thi}}. \tag{5}$$

Then, Eq. (4) reduces to

$$\begin{aligned}\tilde{n}_i &= -\frac{\tau_i\,\tilde{\phi}}{\pi^{1/2}}\int d\zeta\left(1 + \frac{\hat{\omega}}{\zeta - \hat{\omega}}\right)\exp(-\zeta^2)\\ &= -\tau_i\,\tilde{\phi} - \frac{\tau_i\,\tilde{\phi}}{\pi^{1/2}}\int d\zeta\frac{\hat{\omega}}{\zeta - \hat{\omega}}\exp(-\zeta^2).\end{aligned} \tag{6}$$

Equation (6) has singularity at $\hat{\omega} = \zeta$, where ETG mode resonates with the background ions if the phase velocity of ETG mode satisfies $\omega/k_y = V_y$. We can evaluate integral in Eq. (6) by contour integration in complex ζ—plane to obtain

$$\tilde{n}_i = -\tau_i\,\tilde{\phi}\left[1 + i\pi^{1/2}\hat{\omega}\exp(-\hat{\omega}^2)\right]. \tag{7}$$

Similarly, we calculate impurity density perturbation by using drift kinetic theory. The impurity density response is given by

$$\tilde{n}_I = -\tau_I\,\tilde{\phi}\left[1 + i\pi^{1/2}A_I^{1/2}Z\hat{\omega}\exp(-A_I\hat{\omega}^2)\right]. \tag{8}$$

Here, $\tilde{\phi}_k = e\delta\phi_k/T_e$, $\tilde{n}_i = \delta n_i/n_{i0}$, $\tilde{n}_I = \delta n_I/n_{Io}$, $\tau_i = T_e/T_i$, $\tau_I = T_e/T_I$, and $A_I = m_I/m_e$.

Then, we can obtain the perturbed density by substituting of Eqs. (7) and (8) into the quasi-neutrality condition $\tilde{n}_k = \delta n_e/n_{e0} \simeq (n_{i0}/n_{e0})\,\tilde{n}_i + (Zn_{I0}/n_{e0})\,\tilde{n}_I$, giving rise to

$$\begin{aligned}\tilde{n}_k &\approx -\Big[\tau^* + i\tau_i\pi^{1/2}\hat{\omega}\exp(-\hat{\omega}^2)\\ &\quad + \tau_I Z_{eff}\pi^{1/2}A_I^{1/2}\hat{\omega}\exp(-A_I\hat{\omega}^2)\Big]\,\tilde{\phi}_k\\ &= \Lambda_e\,\tilde{\phi}_k.\end{aligned} \tag{9}$$

Here, we define $\tau^* = \tau_i(n_{i0}/n_{e0}) + \tau_I Z_{eff}$, and $Z_{eff} \approx Z^2 n_{I0}/n_{e0}$.

B. Electron dynamics

For electron dynamics, we use the model equations presented in Ref. 15. These equations describe time evolution of perturbed electron density, current, and electron temperature

$$-\frac{d\tilde{n}_k}{dt}-\frac{d\nabla_\perp^2\tilde{\phi}_k}{dt}-\left(1+(1+\eta_e)\nabla_\perp^2\right)\nabla_y\tilde{\phi}_k + \varepsilon_n(\theta)\nabla_y(\tilde{\phi}_k-\tilde{n}_k-\tilde{T}_{ek})+\nabla_{||}\tilde{J}_{||k}=0, \tag{10}$$

$$\frac{d\tilde{J}_{||k}}{dt}-\hat{J}'_{||0}\nabla_y\tilde{\phi}_k+\nabla_{||}(\tilde{\phi}_k-\tilde{n}_k-\tilde{T}_{ek})=0, \tag{11}$$

$$\left(\frac{d}{dt}+\frac{5}{3}\varepsilon_n(\theta)\nabla_y\right)\tilde{T}_{ek}+\left(\eta_e-\frac{2}{3}\right)\nabla_y\tilde{\phi}_k-\frac{2}{3}\frac{d\tilde{n}_k}{dt}=0. \tag{12}$$

Here, $d/dt=\partial/\partial t+\tilde{V}_{E\times B}\cdot\vec{\nabla}$, $\varepsilon_n(\theta)=[\cos\theta+\sin\theta(1/ik_y)\nabla_x]$, $\delta J_{||k}=-en\delta u_{||ek}$, $\tilde{J}_{||k}=\delta J_{||k}/enc_e$ since $\omega/k_{||}\gg c_i$, $v_{*e}=\rho_e c_e/L_n$, $\hat{J}'_{||0}=L_n\nabla_x J_{||0}/enc_e$, c_e is electron thermal velocity. Here, the co-ordinates x, y are normalized to ρ_e, the parallel length scale is normalized to L_n, and the time scale is normalized to L_n/c_e.

III. ELECTRON TRANSPORT IN PEDESTAL

A. Linear theory

One can derive the linear dispersion relation for the ETG mode by Fourier analyzing Eqs. (10)–(12), giving rise to

$$(\tau^*+k_\perp^2)\omega^2+\omega k_y\left[1-\left(1+\frac{10\tau^*}{3}\right)\varepsilon_n-k_\perp^2\left(1+\eta_e+\frac{5\varepsilon_n}{3}\right)\right] + \varepsilon_n k_y^2\left[\eta_e-\frac{7}{3}+\frac{5\varepsilon_n}{3}(1+\tau^*)+\frac{5}{3}(1+\eta_e)k_\perp^2\right] \simeq k_{||}^2\left[\left(1+\frac{5\tau^*}{3}\right)-\left(\eta_e-\frac{2}{3}\right)\frac{k_y}{\omega}\right]-k_{||}k_y\hat{J}'_{||0}. \tag{13}$$

We first solve Eq. (13) in the local limit by treating the parallel compression effect perturbatively (i.e., $\omega\sim\omega_*>k_{||}c_e$ or $k_\theta\rho_e>\varepsilon_n/2q$). For $k_\theta\rho_e>\varepsilon_n/2q$, the leading order solution is $\omega_k=\omega_{r0}+i\gamma_0+\omega_1$ with

$$\omega_{r0}\approx-\frac{k_y}{2(\tau^*+k_\perp^2)}\left[1-\left(1+\frac{10\tau^*}{3}\right)\varepsilon_n-k_\perp^2\rho_e^2\left(1+\eta_e+\frac{5\varepsilon_n}{3}\right)\right] \approx-\frac{k_y}{2(\tau^*+k_\perp^2)}, \tag{14}$$

$$\gamma_0\approx\frac{k_y}{\tau^*+k_\perp^2}[\tau^*\varepsilon_n(\eta_e-\eta_{th})]^{1/2}. \tag{15}$$

The mode becomes unstable when $\eta_e>\eta_{th}$, where $\eta_{th}\approx 2/3+1/4\tau^*\varepsilon_n-1/2\tau^*$. For typical edge pedestal parameters, $R/2L_n=16$, $Z_{eff}=1.5$, and $\eta_{th}\sim 2.1$. The magnitude of the real frequency shift and growth rate due to parallel electron motion are given by

$$\omega_{r1}\simeq\frac{k_{||}^2k_y}{2\tau^*|\omega_{0k}|^2}\left(\eta_e-\frac{2}{3}\right), \tag{16a}$$

$$\gamma_1\simeq-\frac{k_{||}^2}{2\tau^*\gamma_0}\left[\left(1+\frac{5\tau^*}{3}\right)+2\left(\eta_e-\frac{2}{3}\right)(\tau^*+k_\perp^2)\right]+\frac{k_{||}k_y\hat{J}'_{||0}}{2\tau^*\gamma_{0k}}, \tag{16b}$$

Equation (16) shows that parallel electron motion stabilizes the ETG mode while the local current gradient in the pedestal destabilizes it if the condition $k_{||}k_y\hat{J}'_{||0}>0$ is met.

Now, we perform a nonlocal analysis of the linear ETG mode. In order to calculate the radial mode width of the mode in the pedestal, we consider the standard ballooning formalism where perpendicular and parallel wave vectors and curvature terms are represented by $k_x^2+k_y^2=k_y^2(1+(\hat{s}\theta-\alpha\sin\theta))^2$, $k_{||}=-i(\bar{\varepsilon}_n/2q)\partial/\partial\theta$, and $\varepsilon(\theta)=\bar{\varepsilon}_n(\cos\theta+(\hat{s}\theta-\alpha\sin\theta)\sin\theta)$. Here, θ is the usual extended angle co-ordinate in the ballooning formalism, $\bar{\varepsilon}_n=2L_n/R$, $\hat{s}$ is the magnetic shear parameter, q is the safety factor, and $\alpha=q^2\beta_eR/L_P\approx 4q^2\beta_eR/L_n$ is the Shafranov shift parameter. This Shafranov shift parameter turns out to play a crucial role to obtain electron transport scaling in the pedestal, as will be shown shortly.

Using the strong ballooning limit where the mode is localized around $\theta\simeq 0$, one can derive an approximate radial eigen-mode equation

$$\left(A\frac{\partial^2}{\partial\theta^2}+B+C\theta^2\right)\tilde{\phi}_k=0, \tag{17}$$

whose coefficients are given by

$$A=\left(\frac{\bar{\varepsilon}_n}{2q}\right)^2\left[\left(1+\frac{5\Lambda_e}{3}\right)\omega-\left(\eta_e-\frac{2}{3}\right)k_y\right]\Big/\omega,$$

$$B=(\Lambda_e+k_y{}^2)\omega^2+\omega k_y\left[1-k_y{}^2\left(1+\eta_e+\frac{5\bar{\varepsilon}_n}{3}\right)-\left(1+\frac{10\Lambda_e}{3}\right)\bar{\varepsilon}_n\right]+\bar{\varepsilon}_nk_y{}^2\left[\eta_e-\frac{7}{3}+(1+\Lambda_e)\frac{5\bar{\varepsilon}_n}{3}+\frac{5}{3}(1+\eta_e)k_y{}^2\right],$$

$$C\sim k_y{}^2(\hat{s}-\alpha)^2\omega^2.$$

In writing Eq. (17), we expanded finite Larmor radius (FLR) terms as $k_\perp^2=k_y^2[1+(\hat{s}-\alpha)^2\theta^2]$ and neglected the local variation of θ in curvature drift frequency by assuming $k_y^2>(\hat{s}-\alpha-1/2)\,\bar{\varepsilon}_n/(\hat{s}-\alpha)$.

Equation (17) is a parabolic cylinder differential equation whose eigenvalue condition is

$$B=i(2l+1)(AC)^{1/2}. \tag{18}$$

The solution of Eq. (17) is well-known and given by

$$\tilde{\phi}_k\sim H_n(\zeta)\exp(-\zeta^2/2),\quad \zeta=(-C/A)^{1/4}\theta. \tag{19}$$

Here, $H_n(\zeta)$ is the Hermite polynomial of order n, and the condition $\mathrm{Re}\,(-C/A)^{1/2}>0$ must be met to satisfy causality. Then, the corresponding inverse radial width is

$$k_x{}^2\approx k_y{}^2(\hat{s}-\alpha)^2\langle\Delta\theta\rangle^2,\quad \langle\Delta\theta\rangle^2\approx\mathrm{Re}(-A/C)^{1/2}. \tag{20}$$

In the limits $k_y>\bar{\varepsilon}_n/2q$ and $k_y^2<1$, we get the real frequency and growth rate of the ETG mode as given in Eqs. (14) and (15). For $\omega(1+5\tau_i/3)>(\eta_e-2/3)k_y$, we have $A = (\bar{\varepsilon}_n/2q)^2(1+5\tau_i/3)$, the mean width of the eigenmode in ballooning space becomes

$$\langle \Delta\theta \rangle^2 \approx \bar{\varepsilon}_n(1+5\tau^*/3)[\bar{\varepsilon}_n(\eta_e-\eta_{th})/\tau^*]^{-1/2} \times |1+\omega_{0k}/\gamma_{0k}|^{-2}|\hat{s}-\alpha|^{-1}(2qk_y^2)^{-1}. \quad (21)$$

Substituting Eq. (21) into Eq. (20) leads to an expression for the corresponding radial mode width (i.e., $\Delta x \sim k_x^{-1}$),

$$k_x^2 \approx \bar{\varepsilon}_n\,|\hat{s}-\alpha|(1+5\tau^*/3)^{1/2}/2q[\bar{\varepsilon}_n(\eta_e-\eta_{th})/\tau^*]^{1/2}. \quad (22)$$

In this paper, our main objective is to study ETG driven turbulent transport in the context of quasilinear theory. The linear growth rate Eq. (15) and eigenmode width Eq. (22) are then major ingredients to estimate thermal conductivity and fluctuation level (i.e., $e\delta\phi_k/T_e \sim 1/k_xL_T$) of unstable modes. Generally, the quasilinear linear theory is valid if $1/\tau_{relax} < \gamma_k < 1/\tau_{ac}$ (where τ_{relax}, τ_{ac}, and γ_k are the mean field evolution time, the particle auto-correlation time of a field as seen by a particle, and the linear growth rate of the unstable mode, respectively). We refer Ref. 16 to readers for details on the validity of quasilinear theory. We also remark that the short scale and fast growing nature of the ETG mode ($\gamma_k \gg \gamma_{E\times B}$) makes it unaffected by the presence of strong ExB shear in the H-mode pedestal, so as quasilinear particle and thermal fluxes due to ETG mode turbulence.

B. Electron thermal transport in the pedestal

We now evaluate the electron thermal transport scaling in two limiting cases. In both cases, heat transport caused by the magnetic flutter is negligibly small, which agrees with previous numerical[11,17] and theoretical[15,18] studies. The first case is when $\hat{s} > \alpha$. This condition is easily met in low-β, diverted L-mode discharges where $\hat{s}$ is large while α is relatively small. In this case, one can derive

$$(k_x\rho_e)^2 \approx (\hat{s}/2q)[\tau^*\bar{\varepsilon}_n(1+5\tau^*/3)/(\eta_e-\eta_{th})]^{1/2}. \quad (23)$$

This yields the linear mixing length electron thermal diffusion coefficient ($\chi_e = \gamma_k/k_x{}^2$),

$$\chi_e \approx |k_y\rho_e|\frac{q}{\hat{s}}\frac{c_e\rho_e^2}{L_{Te}}. \quad (24)$$

The quasilinear electron thermal conductivity given by Eq. (24) is valid when $\gamma_k > 0$ (i.e., $\eta_e > \eta_{th}$). We note that χ_e in this case represents a standard electron gyro-Bohm scaling. This scaling is identical to the ion gyro-Bohm scaling that has been derived in Ref. 19 for ion thermal conductivity (χ_i) due to ITG turbulence based on full nonlinear considerations. Because of similarity between Eq. (24) and χ_i in Ref. 19, features of χ_i and χ_e in this regime are almost identical; (1) they are inversely proportional to the plasma current via the relation $\chi_e \propto q$ and thermal transport becomes more severe at shorter poloidal wavelength via $\chi_e \propto k_y$.

The striking contrast occurs in the opposite limit, $\hat{s} \le \alpha$. This condition is readily met in pedestal formation phase during which a strong density gradient builds up while keeping the threshold condition$\eta_{th}/\eta_e < 1$. Then, radial wave vector is given by

$$(k_x\lambda_s)^2 \approx 4q(\bar{\varepsilon}_n(\eta_e-\eta_{th})/\tau^*)^{-1/2} \times (1+5\tau^*/3)^{1/2}|1-\hat{s}/\alpha|. \quad (25)$$

This yields the electron thermal conductivity

$$\chi_e \approx \chi_e^{Ohkawa}\frac{|k_y|}{2\tau^*}\frac{(\eta_e-\eta_{th})}{|1-\hat{s}/\alpha|}\frac{1}{(1+5\tau^*/3)^{1/2}}. \quad (26)$$

Here, $\chi_e^{Ohkawa} = c_e\lambda_s{}^2/qR$. Equation (26) contains three notable features. First, it represents that Ohkawa scaling in electron thermal transport can arise in the edge pedestal region where the condition $\hat{s} \le \alpha$ is satisfied. This is the first analytic derivation of the appearance of Ohkawa scaling in electron thermal transport due to ETG turbulence. Our result also indicates that the Ohkawa scaling can originate from *electrostatic* ETG turbulence combined with the geometrical effect. In our work, the finite electron β effect in χ_e is embodied via the Shafranov shift parameter α in the ballooning formulation. This implies that the Ohkawa scaling can be realized in linear electrostatic ETG theory without invoking the nonlinear inverse cascade process.

It is of interest to compare Eq. (26) to neoclassical ion thermal diffusivity in the pedestal. Taking ion thermal diffusivity in the plateau regime, $\chi_i^{neo} \sim qc_i\rho_i{}^2/R$, we find that the ratio χ_i^{neo}/χ_e is given by $\chi_i^{neo}/\chi_e \sim (\alpha R/8L_n)(m_i/m_e)^{1/2} \le 1$,which is smaller than or comparable to unity depending on edge pedestal conditions. Thus, turbulent electron thermal conduction will have a larger contribution than that of ions in the edge pedestal if ion turbulent transport is fully suppressed. This is consistent with the experimental observations made at ASDEX Upgrade, showing χ_e is three to ten times higher than χ_i inside the pedestal.[20]

Second, χ_e is proportional to a *local* plasma current, i.e., $\chi_e \propto 1/q$. Combined with the Ohkawa scaling, one notices that χ_e is proportional to the ratio of pedestal Greenwald density to the pedestal density, i.e., $\chi_e \propto I_p/na^2$, where I_p and $n$are the plasma current and density in the pedestal. Thus, Eq. (26) predicts that the pedestal electron thermal confinement follows the neo-Alcator scaling. A possible implication of this pedestal confinement scaling is that τ_E in the H-mode will have stronger density dependence compared to the L-mode when it is expressed in terms of empirical scaling law. This tendency is consistent with the empirical L-mode and H-mode energy confinement scalings reported in Refs. 21 and 22, respectively, showing an increase of density exponent in τ_E scaling.

The other implication of the pedestal confinement scaling is that electron confinement will deteriorate as a bootstrap current builds up in the pedestal region. Increase of χ_e implies the increase of the turbulence amplitude, resulting in the generation of anomalous electron viscosity ($\mu_{||e}$). Since this turbulence-driven $\mu_{||e}$ (hyper-resistivity) is likely to accelerate magnetic reconnection and ensuing ELM crash,[23] ETG driven residual turbulence may enhance ELM activities. To realize an ELM crash in Ref. 23, they used $\mu_{||e}$ as the same value of χ_e without theoretical justification. In our formulation, it is straightforward to obtain $\mu_{||e}$ and χ_e by calculating the radial current flux and electron heat flux, resulting in

$$\mu_{e||} = m_e\langle\tilde{v}_{rk}(n_0\tilde{J}_{||e-k} + \tilde{n}_{-k}J_{||0})\rangle \simeq m_e n_0[-\mu_{||e}dJ_{||0}/dr + V_{pinch}J_{||0}], \quad (27)$$

$$Q_e = \langle\tilde{v}_{rk}(n_0\tilde{T}_{e-k} + \tilde{n}_{-k}T_{e0})\rangle \approx n_0[-\chi_e dT_e/dr + V_{pinch}T_e]. \quad (28)$$

Here, $\mu_{||e}$ is found to be equal to the value of χ_e, as in the case of ITG turbulence[24] and given by

$$\mu_{||e} \sim \chi_e \approx c_e L_n \frac{k_\theta^2\rho_e^2\gamma_{0k}}{|\omega_{0k}|^2}\left|\frac{e\phi_k}{T_e}\right|^2. \quad (29)$$

By using the mixing length $|\tilde{\phi}_k| \sim 1/k_x L_T$ and Eq. (25), $\mu_{||e}$ and χ_e become

$$\mu_{||e} = \chi_e \approx \chi_e^{ohkawa}|k_y\rho_e|\eta_e^2(\eta_e - \eta_{th}). \quad (30)$$

One may think that $J_{||0}$ in Eq. (27) arises from a bootstrap current in the pedestal region. Equation (30) shows that ETG turbulence yields $\chi_e = \mu_{||e}$ (i.e., electron Prandtl number due to ETG turbulence is 1) in the quasilinear limit. Thus, one can interpret that $\mu_{||e}$ used in Ref. 23 actually originates from ETG turbulence, providing a theoretical justification of the assumption made in Ref. 23. This exemplifies the importance of ETG turbulence in edge pedestal dynamics.

Third, we note that χ_e is inversely proportional to the parameter $|1-\hat{s}/\alpha|$. This indicates a possible jump of electron thermal transport as α approaches to $\hat{s}$. Even though small but finite resistivity, which is not considered in this paper, will prevent such a blow up in electron thermal transport, a significant increase of χ_e when $\hat{s} \sim \alpha$ is an unavoidable consequence of our theory. For instance, if we fix all parameters and perform $\hat{s}$ scans, χ_e will first increase gradually and show a big jump as α approaches to $\hat{s}$. This tendency reproduces features observed in gyrokinetic ETG simulations[12,14] showing a big jump of $\chi_e \sim 10\chi_{eGB}$ in shear scans without noticeable changes of linear mode characteristics.

C. ETG driven particle pinch

The non-adiabaticity of ions and impurities in the edge pedestal region (Eq. (9)) can induce particle flux by providing the phase shift between density and potential fluctuations. The particle flux, calculated from $\Gamma_n \equiv \langle\tilde{v}_{rk}\tilde{n}_{-k}\rangle$, is given by

$$\Gamma_n \approx \pi^{1/2}\tau_i n c_e k_y\rho_e\left(\frac{\omega_r}{k_\perp V_{thi}}\right)\left[\exp\left(-\frac{\omega_r^2}{k_\perp^2 V_{thi}^2}\right) + \frac{\tau_I}{\tau_i}Z_{eff}A_i^{1/2}\exp-\left(-A_i^{1/2}\frac{\tau_I}{\tau_i}\frac{\omega_r^2}{k_\perp^2 V_{thi}^2}\right)\right]|\tilde{\phi}_k|^2. \quad (31)$$

For $|\tilde{\phi}_k| \sim 1/k_x L_T$, where k_x is given by Eq. (25), Eq. (31) can be written as

$$\Gamma_n \approx -\pi^{1/2}n|k_y\rho_e|\left(\frac{\chi_e^{Ohkawa}}{4L_{Te}}\right)\left(\frac{\tau_i}{\tau^*}\right)^{3/2}\left(\frac{R}{L_{Te}}\right)^{1/2} \times \left(\frac{\rho_s}{L_{Te}}\right)\left(\frac{1-\eta_{th}/\eta_e}{1+5\tau^*/3}\right)^{1/2}\frac{1}{|1-\hat{s}/\alpha|} \times (\exp(-\hat{\omega}^2) + (T_i/T_I)Z_{eff}A_I^{1/2}\exp(-A_I\hat{\omega}^2)). \quad (32)$$

Here, $A_I = m_I/m_i$. Equation (32) represents an ETG driven thermoelectric pinch in the pedestal. In the absence of recycled neutral flux entering into the pedestal from wall, a steady state (i.e., $\Gamma_n = 0$) is set by the condition $\eta_e \sim \eta_{th} \sim 2$. The density scale length L_n locked to L_{Te}, which is set by ETG heat balance $\langle L_{Te}\rangle \sim Q_e/\chi_e T_e$, where $\langle L_{Te}\rangle$ is the mean pedestal temperature scale length and Q_e is the heat flux entering into pedestal from the core.

ETG driven particle pinch may have an influence on the rapid formation of an edge density pedestal, as pointed out in Ref. 25 where density pedestal formation time is calculated based on inward pinch driven by poloidally asymmetric electric field. In a similar vein, we calculate the pinch time (pedestal formation time) due to ETG turbulence as $\tau = nL_n/\Gamma_n$. For the purpose of rough estimate, we calculate τ for two sets of representative tokamak plasma parameters: $n = 0.2, 0.7$, $T_e = 0.8, 2.5$, $B = 2, 5$, $R = 1.5, 6$, $\eta_e = 2.0$, $Z_{eff} = 2.0$, $\hat{s} = 2$, $\alpha = 2.5$, $q = 3$, $\tau_i = \tau_I = 1$, $A_I = 6$, $A_i = 2$, $\langle k_y\rho_e\rangle = 0.5$, $L_n/a = 0.04$ (where density in $10^{20}\mathrm{m}^{-3}$, temperature in KeV, and magnetic field in tesla), resulting in $\tau = 0.1$, 20 ms for each case. This showed that the density pedestal formation occurs within a 100 μs in present-day medium size tokamaks; whereas in large machines like ITER, the pedestal formation time will be slower and typically $\tau \leq 20$ ms.

A scenario for the acceleration of density pedestal formation can be summarized as follows. Ion temperature and density pedestals start to form first as ρ_i scale turbulence is quenched by $E\times B$ shear. Since ETG turbulence will still be active in this circumstance, it will drive inward particle pinch, accelerating the density pedestal formation. This pedestal formation continues until it hits the ETG threshold $\eta_e \sim \eta_{th} \sim 2$. This predicts the pedestal electron temperature profile must remain near the ETG threshold value, as observed in ASDEX-U experiments.[26]

IV. CONCLUSIONS

To summarize, we have presented a theory of electron turbulent transport driven by electrostatic ETG turbulence in the edge pedestal region. A summary of the main results of this paper is as follows:

(i) The electron thermal conductivity exhibits a different scaling depending on relative values of magnetic shear ($\hat{s}$) vs. the Shafranov shift parameter (α). It exhibits electron gyro-Bohm-like scaling when $\hat{s} > \alpha$, while follows the Ohkawa scaling (i.e., electron skin depth size scaling in radial correlation length) when $\hat{s} < \alpha$. This Ohkawa scaling governs electron thermal transport in the edge pedestal region of H-mode plasmas.

(ii) ETG turbulence can induce an inward particle pinch during the development phase of an edge pedestal due to the non-adiabatic nature of ions and impurities. This can lead to the rapid (compared to energy confinement time) formation of density pedestal until it hits the ballooning mode (BM) limit. Our theory naturally predicts that the pedestal electron temperature profile must remain near the ETG threshold value.

An important message of this paper is that the Ohkawa scaling in electron thermal transport can be derived using a linear electrostatic ETG theory when one takes Shafranov shift effects into account. This prediction may be checked in existing codes by including the Shafranov shift effect consistently. Our theory also has some experimental implications. Among them, we pointed out the possible acceleration of density pedestal formation and the restoration of Alcator scaling in the pedestal region. The persistence of η_e near to η_{th} in the pedestal region, which is predicted in this paper, has been observed in some tokamak experiments.[24] More detailed experimental investigation of density pedestal formation process in these lines would be interesting to validate the present theory.

It is instructive here to discuss possible implications of the co-existence of KBM and ETG turbulences. ETG turbulence will prevail in pedestal (as long as $\eta_e > \eta_{th}$) no matter whether the KBM threshold condition is met or not. KBMs, on the other hand, become unstable when a pressure gradient exceeds some threshold and have a longer spatial scale ($k_\perp{}^{-1} > \rho_i$) compared to shorter scale ($\rho_i > k_\perp{}^{-1} > \rho_e$) ETG modes. Once the KBM onset condition is met, ETG and KBM turbulences may co-exist in the pedestal. Because of their disparate spatial scales, they will affect to pedestal profile dynamics in a different way. First, KBM turbulence may modify the pressure profile on a rather longer scale. The shorter scale ETG modes will react by this change of pressure profile through the multi-scale interaction, and possibly be modulated by the KBM modes. How this multi-scale interaction happens and what would be the consequence of this have not fully elucidated yet. The physics of this multi-scale interaction is beyond the scope of this paper and will be an interesting future subject.

We plan to extend this analysis to include the interaction between ETG and BMs to study back reaction of BM to ETG via multi-scale interaction. We also plan to investigate nonlinear saturation of ETG turbulence by generating electron geodesic acoustic modes. These will be published in the future. Finally, we remark that our theory and discussions are based on the assumption that particle transport comes entirely from the ETG mode when TEM is completely suppressed in the edge pedestal. In actual experiments, however, this ETG turbulence will co-exists with residualρ_i scale turbulence, which may affect the pedestal dynamics. To draw more concrete conclusions, it will be necessary to perform both experimental fluctuation measurements spanning low to high k with good spatial resolution and simulations taking ETG and BM dynamics self-consistently.

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